# CLIQUE COVER

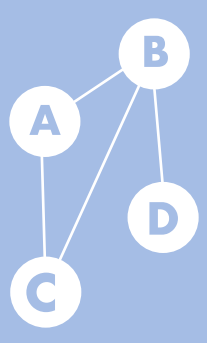
By Kate Bagshaw

## *What is a clique cover?*

The clique cover problem is an NP-Complete problem and can include finding the maximum clique, the maximum weight clique, or the maximal clique.

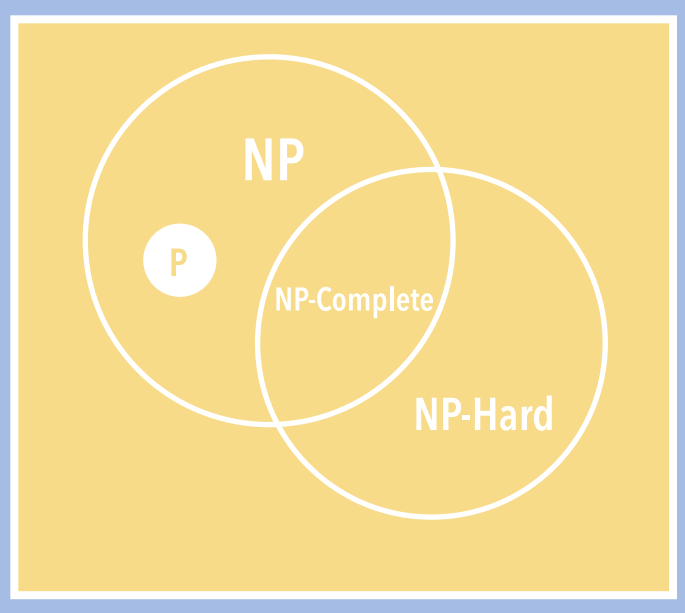
## *What is a clique?*

A clique is a subset of a graph where any two vertices in the graph are adjacent & connected by an edge.




Cliques:
{A,B,C}, {B,D},
{A,B}, {B,C}, {A,C}

Not Cliques:
{A,B,C,D}, {C,D},
{A,D}



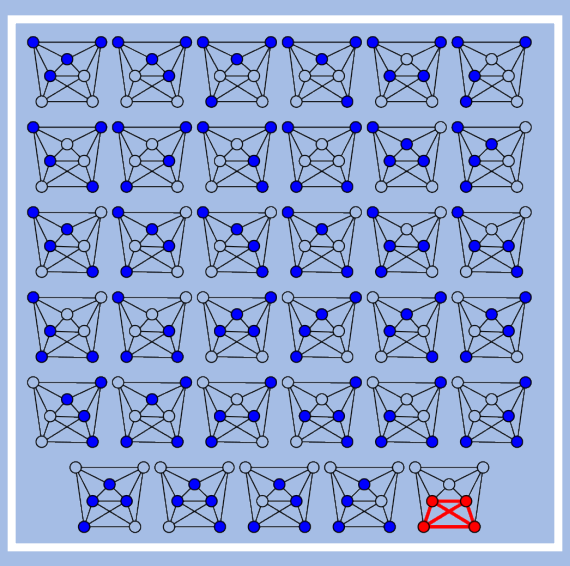

*NP-Complete* problems are in both *NP* and *NP-Hard*, and therefore their solutions can be checked in polynomial time, but they cannot be solved in polynomial time.

# *How do you solve the clique cover problem?*

## Brute Force Algorithm:

* Systematically check *each possible combination* of vertices until a clique of size k is found.

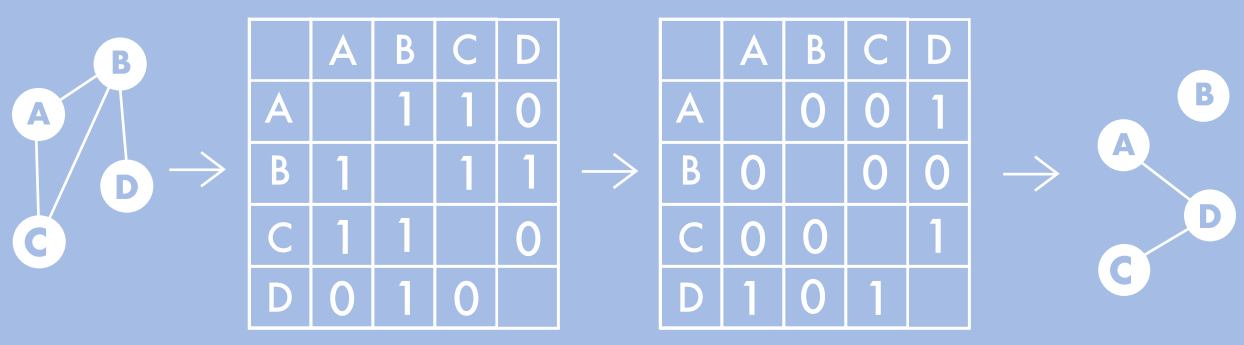
Example:



Brute force is correct, but **extremely computationally expensive!**
Finding a size 4 clique in a graph with 7 vertices takes 35 iterations!

## Reducing into Independent Set:

* Flip bits in adjacency matrix and construct new inverse graph




| | A | B | C | D |
|---|---|---|---|---|
| A | | 1 | 1 | 0 |
| B | 1 | | 1 | 1 |
| C | 1 | 1 | | 0 |
| D | 0 | 1 | 0 | |

| | A | B | C | D |
|---|---|---|---|---|
| A | | 0 | 0 | 1 |
| B | 0 | | 0 | 0 |
| C | 0 | 0 | | 1 |
| D | 1 | 0 | 1 | |

The independent set {A,B,C} in the inverse graph is the clique in the original graph. Since we know the independent set problem is NP-Complete, the Clique Cover problem must then be NP-Complete!

# *Applications*

## Social networking:

People can be viewed as vertices and the edges between them can represented as relationships in an undirected graph.

* *Clique covers can be used to find friend groups where everyone knows each other within a large group of people.*

## *References*


- *https://en.wikipedia.org/wiki/NP-completeness*
- *https://en.wikipedia.org/wiki/Clique_cover*
- *Graph diagram: https://en.wikipedia.org/wiki/Clique_problem#History_and_applications*
- *Facebook logo adapted from: http://chittagongit.com/icon/facebook-logo-icon-5.html*